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ON AN INFINITE DIMENSIONAL GENERALIZATION OF THE EXCESS DEMAND THEOREM OF DAVID GALE

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Abstract. [9] proved the excess demand theorem (known as the Gale-Nikaido-Debreu theorem) by means of the Knaster-Kuratowski-Mazurkiewicz (KKM) lemma. We generalize the excess demand theorem to infinite dimensional spaces by adopting the infinite dimensional extension of the KKM lemma by [8].

1. Introduction

One of the fundamental problems in economic theory is the existence of a competitive equilibrium. Perhaps this is one of the deepest problems in economic theory as it is mathematically demanding.

The first attempt to prove the existence of an equilibrium was made by [18] in his revolutionary book "Elements of Pure Economics" that introduced the neoclassical economic theory that we study today. However, the existence proof of Walras was at best incomplete. It was [17] who provided a rigorous existence proof but his proof was focused on a closed production model rather than a competitive equilibrium for an economy. The first existence of equilibrium proofs for an economy were given by [14] and a few months later by [3]. Both proofs were based on fixed point theorems. Specifically, McKenzie's proof was based on a clever mapping whose Kakutani fixed point yielded an equilibrium (that was the first application of the Kakutani fixed point theorem in economics). Arrow and Debreu had a different approach. Specifically, they converted the economy into a game (or social system as it was called by [6]) and by appealing to an existence theorem of [6] they showed that the game has a non-cooperative equilibrium à-la Cournot-Nash. Then they demonstrated that the existence of a non-cooperative equilibrium for the game implies the existence of a competitive equilibrium for the economy. The Eilenberg and Montgomery fixed point theorem was used in the Debreu paper and a fortiori in [3] as it was Debreu's existence theorem that was applied.

Subsequently, [9], [15], [7], and [13] all provided proofs of the so-called "excess demand theorem" or "Gale-Nikaido-Debreu (GND) theorem". The idea is to describe the whole economy by an excess demand correspondence and show that there is an equilibrium, i.e., a price vector that clears the markets (see Section 2 of this paper). Gale's proof is based on the Knaster-Kuratowski-Mazurkiewicz lemma (KKM

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lemma), Nikaido and Debreu provided similar proofs based on the Kakutani fixed point theorem, and Kuhn's proof is based on the Eilenberg and Montgomery fixed point theorem.

It should be noted that all those proofs are finite dimensional. It is our purpose to generalize the excess demand theorem to infinite dimensional spaces. Specifically, we will follow the Gale approach and employ the infinite dimensional generalization of the KKM lemma due to [8]. Our infinite dimensional version not only gives as a corollary the excess demand theorem of Gale, Nikaido, Debreu, and Kuhn, but it is also more general as three of the main assumptions needed are weakened (see Section 3).

The paper proceeds as follows. Section 2 describes the mathematical preliminaries and the economy. Section 3 presents the theorem. Section 4 provides concluding remarks.

2. Model

2.1. **Notation and Definitions.** We let 2^A denote the set of all subsets of A; $\mathbb{R}^\ell$ denotes the ℓ-fold Cartesian product of the set of real numbers $\mathbb{R}$ and $\mathbb{R}^\ell_+$ denotes the closed positive orthant of $\mathbb{R}^\ell$; $\text{int}A$ denotes the interior of the set A, $\text{cl}A$ denotes its closure, and $\text{con}A$ denotes its convex hull; $A \setminus B := \{x \in A : x \in A \text{ and } x \notin B\}$ denotes the set subtraction. If X is a linear topological space, its topological dual is the space X' of all continuous linear functionals on X; if $q \in X'$ and $y \in X$, the value $q(y)$ of q at y is also denoted by $q \cdot y$; if C is a subset of X, we let $C^\circ = \{p \in X' : p \cdot x \leq 0 \text{ for all } x \in C\}$ be the polar cone of C.

Let X be a topological space and let Y be a linear topological space. A correspondence ψ: $X \to 2^Y$ is said to be **upper semi-continuous** (u.s.c.)[1] if the set $\{x \in X : \psi(x) \subseteq V\}$ is open in X for any open subset V of Y. A correspondence ψ: $X \to 2^Y$ is said to be **upper demi-continuous** (u.d.c.) if the set $\{x \in X : \psi(x) \subseteq V\}$ is open in X for any open half space V of Y. Clearly an u.s.c. correspondence is also u.d.c.. It can be easily checked that the reverse is not true. The correspondence ψ: $X \to 2^Y$ is said to have **open lower sections** if for all $y \in Y$, the set $\psi^{-1}(y) = \{x \in X : y \in \psi(x)\}$ is open in X. If ψ has open lower sections, then it is **lower semi-continuous**, i.e., the set $\{x \in X : \psi(x) \cap V \neq \emptyset\}$ is open in X for any open subset V of Y.

2.2. **Economy and Equilibrium.** An **economy** $\mathcal{E}$ is a set of triples $\mathcal{E} = \{(X_i, u_i, e_i) : i \in I\}$ where:

- I is a finite set of **agents**, i.e., $I = \{1, 2, ..., n\}$,
- $X_i \subseteq \mathbb{R}^\ell$ is the **consumption set** of agent i,
- $u_i : X_i \to \mathbb{R}$ is the **utility function** of agent i, and
- $e_i \in X_i$ is the **initial endowment** of agent i.

Let $\Delta^{\ell-1} = \{p \in \mathbb{R}^\ell_+ : \sum_{l=1}^{\ell} p^l = 1\}$ denote the **price set**. The **budget set** of agent i is a correspondence $B_i : \Delta^{\ell-1} \to 2^{\mathbb{R}^\ell}$ defined by $B_i(p) = \{x_i \in X_i : p \cdot x_i \leq p \cdot e_i\}$, that is, the set of admissible consumptions whose value, i.e., $p \cdot x_i$, cannot exceed the value of the initial endowment (resources), i.e., $p \cdot e_i$.

[1]Sometimes, upper semi-continuous is also called upper hemi-continuous.

The **demand set** of agent i is the set of optimal consumptions in the budget set, i.e., the demand set is a correspondence $D_i : \Delta^{\ell-1} \to 2^{\mathbb{R}^\ell}$ defined as

$$D_i(p) = \{x_i \in B_i(p) : u_i(x_i) \geq u_i(y_i),\ \forall y_i \in B_i(p)\}.$$

The **excess demand** correspondence ζ_i for agent i is defined by $\zeta_i(p) = D_i(p) - \{e_i\}$, and the **aggregate excess demand** $\zeta : \Delta^{\ell-1} \to 2^{\mathbb{R}^\ell}$ is defined by $\zeta(p) = \sum_{i\in I} \zeta_i(p)$.

An **equilibrium** for the economy $\mathcal{E} = \{(X_i, u_i, e_i) : i \in I\}$ is a consumption-price pair $(x^*, p^*) \in \prod_{i\in I} X_i \times \Delta^{\ell-1}$ such that:

(i) $x_i^* \in D_i(p^*)$ for all $i \in I$,
(ii) $\sum_{i\in I} x_i^* \leq \sum_{i\in I} e_i$.

Equivalently, we say that the economy $\mathcal{E}$ has an **equilibrium** if there exists a price $p^* \in \Delta^{\ell-1}$ such that $\zeta(p^*) \cap (-\mathbb{R}_+^\ell) \neq \emptyset$.

In this sequel we will describe the economy by an excess demand correspondence $\zeta : \Delta^{\ell-1} \to 2^{\mathbb{R}^\ell}$. This is the approach originated by Gale, Nikaido, Debreu, and Kuhn. Although we consider in this paper a finite set of consumers, the excess demand theorem can also apply to infinitely many consumers. In particular, in the above setting, one could replace the finite set I with an atomless measure space, and also replace the sum with an integral. See for example [12].

2.3. **Finite Dimensional Excess Demand Theorem.** [9] and subsequently [15], [7], and [13] proved the following finite dimensional fundamental result in general equilibrium theory.

Theorem 2.1 (Excess Demand Theorem)**.** *Let $\Delta^{\ell-1} = \{p \in \mathbb{R}_+^\ell : \sum_{l=1}^{\ell} p_l = 1\}$ be the price simplex in $\mathbb{R}^\ell$, and $\zeta : \Delta^{\ell-1} \to 2^{\mathbb{R}^\ell}$ be an excess demand satisfying the following assumptions:*

(i) *the correspondence ζ is u.s.c.,*
(ii) *for all $p \in \Delta^{\ell-1}$, $\zeta(p)$ is nonempty, compact, and convex,*
(iii) *(Walras' Law) for every $p \in \Delta^{\ell-1}$ and for every $z \in \zeta(p)$, $p \cdot z \leq 0$.*

*Then, there exists an **equilibrium**, i.e., there exists $p^* \in \Delta^{\ell-1}$ such that $\zeta(p^*) \cap (-\mathbb{R}_+^\ell) \neq \emptyset$.*

Gale's proof of the above theorem is based on the Knaster-Kuratowski-Mazurkiewicz lemma (KKM lemma), Nikaido and Debreu provided similar proofs based on the Kakutani fixed point theorem, and Kuhn's proof is based on the Eilenberg and Montgomery fixed point theorem.

3. An infinite dimensional generalization of the Gale excess demand theorem

3.1. **Infinite Dimensional Excess Demand Theorem.** This section provides an infinite dimensional generalization of the previous excess demand theorem (Theorem 2.1), whose proof will rely on the KKMF lemma, an infinite dimensional generalization of the KKM lemma due to [8].

We now state our main result.

Theorem 3.1. *Let X be a Hausdorff locally convex linear topological space, $C \subseteq X$ a closed, convex cone (of vertex 0) such that $C \neq X$ and* $\text{int}C \neq \emptyset$. *For $v \in \text{int}C$, define $\Delta = \{p \in C^\circ : p \cdot v = -1\}$ and let $\zeta : \Delta \to 2^X$ be an excess demand correspondence such that:*

(i) *$\zeta : \Delta \to 2^X$ is weak* u.d.c., i.e., ζ is u.d.c. with $\Delta \subseteq X'$ endowed with the relative weak* topology of X', and X endowed with its initial Hausdorff topology;*

(ii) *for each $p \in \Delta$, $\zeta(p)$ is nonempty, closed, and convex;*

(iii) *(Weak Walras' Law) for all $p \in \Delta$, there exists $z \in \zeta(p)$, such that $p \cdot z \leq 0$.*

Then, there exists $p^* \in \Delta$ such that $0 \in \text{cl}\big[\zeta(p^*) - C\big]$.

Moreover, if we additionally assume that the excess demand ζ is compact valued, then there exists an ***equilibrium****, i.e., $p^* \in \Delta$ such that $\zeta(p^*) \cap C \neq \emptyset$.*

We point out that Theorem 3.1 also generalizes the finite dimensional result of Gale-Nikaido-Debreu-Kuhn as (i) our excess demand correspondence is only assumed to be *u.d.c.* (a weaker condition than u.s.c.), (*ii*) Walras' law is stated in a *weaker form,* in the sense that, for all $p \in \Delta$, $p \cdot z \leq 0$ for some $z \in \zeta(p)$ (instead of for all $z \in \zeta(p)$), and (*iii*) we show the existence of a price p^* satisfying a weaker form of equilibrium when the excess demand correspondence is *not assumed to be compact valued.*

The proof of the theorem is given in the next section and it relies on the following infinite dimensional generalization of the KKM lemma, due to [8], that will be called hereafter the KKMF lemma.

Lemma 3.2 (KKMF Lemma)**.** *Let Y be a nonempty subset in a Hausdorff linear topological space V and $F : Y \to 2^V$ satisfy:*

(i) *$F(x)$ is closed for* **all** *$x \in Y$ and $F(x)$ is compact for* **some** *$x \in Y$, and*

(ii) *the convex hull of any finite subset $\{x_1, ..., x_n\}$ of Y is contained in $\bigcup_{i=1}^n F(x_i)$.*

Then, $\bigcap_{x \in Y} F(x) \neq \emptyset$.

A proof of the KKMF lemma can be found in [8]. See also Section 4 for the equivalence between the KKMF lemma and a fixed point theorem due to [4].

3.2. **Proof of Theorem 3.1.** The proof of the theorem is a consequence of the following claims.

• **Claim 1.** The set Δ is nonempty and weak* compact.

Proof. For the weak* compactness we refer to James (1970). We now prove that Δ is nonempty. Since C is a closed convex cone (of vertex 0) that is not the whole space, we deduce that $C^o \neq \{0\}$ from the bipolar theorem. Thus, we can choose $q^* \in C^\circ$, $q^* \neq 0$ and it suffices to prove that $q^* \cdot v < 0$ (where $v \in \text{int}C$ by assumption), which implies that $q^*/ - q^* \cdot v \in \Delta$.

We now prove that $q^* \cdot v < 0$. Indeed, there exists $e \in X$ such that $q^*(e) \neq 0$ (since $q^* \neq 0$) and we can additionally suppose that $q^* \cdot e > 0$. Thus, for $t > 0$ small enough, $v + te \in C$ (since $v \in \text{int}C$), hence $q^* \cdot (v + te) \leq 0$ (since $q^* \in C^\circ$). Thus, $q^* \cdot v \leq -tq^* \cdot e < 0$. □

• **Claim 2.** $\exists p^* \in \Delta,\ \forall q \in \Delta,\ \exists z \in \zeta(p^*),\ q \cdot z \leq 0.$

Proof. Define the correspondence $F : \Delta \to 2^{\Delta}$ by:

$$F(q) := \{p \in \Delta : \exists z \in \zeta(p),\ q \cdot z \leq 0\} \text{ for } q \in \Delta.$$

Then Claim 2 is equivalent to:

$$\exists p^* \in \Delta,\ p^* \in \bigcap_{q \in \Delta} F(q).$$

The existence of such a p^* is a consequence of the KKMF lemma and we only need to prove that all assumptions of KKMF lemma are satisfied by the correspondence F.

(i) We first show that, for each $q \in \Delta$, $F(q)$ is weak* closed in Δ. To see this, let $q \in \Delta$, then $V_q = \{x \in X : q \cdot x > 0\}$ is an open half space in X, thus the set $\Delta \setminus F(q) = \{p \in \Delta : \zeta(p) \subseteq V_q\}$ is weak* open in Δ since ζ is weak* u.d.c.. Thus $F(q)$ is weak* closed in Δ.

Moreover, for all $p \in \Delta$, $F(p)$ is weak* compact, since $F(p)$ is a weak* closed subset of Δ, which is weak* compact by Claim 1.

(ii) We now prove that for any set of points $\{q_1, ..., q_n\} \subseteq \Delta$, $\text{con}\{q_1, ..., q_n\} \subseteq \bigcup_{\iota=1}^{n} F(q_\iota)$. Suppose otherwise that there exists $q \in \text{con}\{q_1, ..., q_n\}$, that is, $q = \sum_{\iota=1}^{n} \lambda_\iota q_\iota$ for some $\lambda_\iota \geq 0$, such that $\sum_{\iota=1}^{n} \lambda_\iota = 1$, and $q \notin \bigcup_{\iota=1}^{n} F(q_\iota)$, that is, $q \in \bigcap_{\iota=1}^{n} \Delta \setminus F(q_\iota)$. This implies that $\zeta(q) \subseteq V_{q_\iota} := \{x \in X : q_\iota \cdot x > 0\}$ for all ι, that is, for all $z \in \zeta(q)$ one has $q_\iota \cdot z > 0$ for all ι, and hence, $q \cdot z = \sum_{\iota=1}^{n} \lambda_\iota q_\iota \cdot z > 0$. But this contradicts the Weak Walras' Law (Assumption (iii)).

• **Claim 3.** $0 \in \text{cl}\big[\zeta(p^*) - C\big]$ for some $p^* \in \Delta$.

Proof. We let p^* as in Claim 2 and we prove that $0 \in \text{cl}\big[\zeta(p^*) - C\big]$ by contradiction. Suppose that $0 \notin \text{cl}\big[\zeta(p^*) - C\big]$. Then the point 0 and the nonempty, closed, convex set $\text{cl}\big[\zeta(p^*) - C\big] \subseteq X$ can be strictly separated by a continuous linear functional, i.e., there exists $q^* \in X'$, $q^* \neq 0$ such that:

$$0 = q^* \cdot 0 < \inf_{x \in \text{cl}[\zeta(p^*) - C]} q^* \cdot x \leq \inf_{z \in \zeta(p^*),\, c \in C} q^* \cdot (z - c).$$

Thus one has:

$$a := \sup_{c \in C} q^* \cdot c < \inf_{z \in \zeta(p^*)} q^* \cdot z =: b.$$

But $a = 0$ since C is a cone (of vertex 0). Thus, $\sup_{c \in C} q^* \cdot c \leq 0$, that is, $q^* \in C^\circ$. Recalling that $q^* \neq 0$ we deduce that $q^* \cdot v < 0$ (as already shown in the proof of Claim 1). We let $\lambda := -q^* \cdot v > 0$ and, from above, we deduce that $(q^*/\lambda) \in \Delta$. Consequently, from Claim 2, for $q := (q^*/\lambda) \in \Delta$, there exists $z^* \in \zeta(p^*)$ such that $(q^*/\lambda) \cdot z^* \leq 0$. Thus

$$b := \inf_{z \in \zeta(p^*)} q^* \cdot z \leq q^* \cdot z^* \leq 0.$$

Then $b \leq 0$ contradicts the above inequality $0 = a < b$. □

• **Claim 4.** If $\zeta(p^*)$ is additionally assumed to be compact, then:

$$0 \in \text{cl}\big[\zeta(p^*) - C\big] \quad \Leftrightarrow \quad 0 \in \zeta(p^*) - C \quad \Leftrightarrow \quad \zeta(p^*) \cap C \neq \emptyset.$$

Proof. The first equivalence follows from the fact that the set $\zeta(p^*) - C$ is closed since $\zeta(p^*)$ is compact and C is closed. To prove the second equivalence, notice that $0 \in \zeta(p^*) - C$ if and only if $0 = z - c$ for some $(z, c) \in \zeta(p^*) \times C$ if and only if $z \in \zeta(p^*) \cap C$ for some $z \in X$. □

4. Concluding remarks

The first remark shows that Theorem 3.1 can be proved as a consequence of [4] fixed point theorem, following [21] who proved that the KKMF lemma and the Browder fixed point theorem are equivalent in the sense that each one can be derived from the other.[2]

Theorem 4.1. *Let Y be a nonempty, compact, convex subset of a Hausdorff linear topological space and $\phi : Y \to 2^Y$ be a convex and nonempty valued correspondence with open lower sections. Then ϕ has a fixed point, i.e., there exists $x^* \in Y$ such that $x^* \in \phi(x^*)$.*

The Browder fixed point theorem can be equivalently stated as follows.

Theorem 4.2. *Let Y be a nonempty, compact, convex subset of a Hausdorff linear topological space and $\phi : Y \to 2^Y$ be a convex valued correspondence with open lower sections such that $x \notin \phi(x)$ for all $x \in Y$. Then there exists $x^* \in Y$ such that $\phi(x^*) = \emptyset$.*

Remark 4.3. In the proof of Theorem 3.1, one can replace the KKMF lemma with the Browder fixed point theorem to prove Claim 2 and the rest of the proof is unchanged:

(Claim 2:) $\exists p^* \in \Delta,\ \forall q \in \Delta,\ \exists z \in \zeta(p^*), q \cdot z \leq 0.$

Alternative Proof of Claim 2. Define the correspondence $\phi : \Delta \to 2^\Delta$ by:

$$\phi(p) = \{q \in \Delta : q \cdot z > 0, \forall z \in \zeta(p)\}.$$

Notice that the statement of Claim 2 is equivalent to saying that $\phi(p^*) = \emptyset$ for some $p^* \in \Delta$. Thus, we will deduce Claim 2 from the Browder fixed point theorem (Theorem 4) and we only need to check that all its assumptions are satisfied. Indeed, first $p \notin \phi(p)$ for all $p \in \Delta$; otherwise we get a contradiction with the Weak Walras' Law (Assumption (iii)). Second, ϕ is clearly convex-valued. Finally, for all $q \in \Delta$,

$$\phi^{-1}(q) := \{p \in \Delta : q \in \phi(p)\} = \{p \in \Delta : \zeta(p) \subseteq V_q\}$$

with $V_q := \{z \in X \ :\ q \cdot z > 0\}$ is weak* open in Δ since V_q is an open half-space and ζ is weak* u.d.c.. □

Remark 4.4. Alternative infinite dimensional proofs of the excess demand theorem have been obtained by [1] and [20]. The Aliprantis-Brown proof used an approximation argument and it is stated in terms of excess demand functions instead of correspondences. The proof by [20] makes use of the Tychonoff fixed point theorem. As shown in [20], the [1] is a corollary of [20] and a fortiori a corollary of our Theorem 3.1.

Remark 4.5. Recent works by [10], [5], [11], [2], and [16] have shown that the continuity assumption on the excess demand theorem can be further weakened to the continuous inclusion property. As in [10], one can show that the proof of Theorem 3.1 can be modified to allow for an excess demand correspondence satisfying the continuous inclusion property. We hope to take up those details in a subsequent paper.

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[2]See also [19] for an alternative proof and extension of the Browder fixed point theorem.

□

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